

Calc BC Gold #4 - Solutions

I - No Calc

$$\textcircled{1} \lim_{h \rightarrow 0} \frac{1}{h} \int_a^{a+h} f'(x) dx$$

$$= \lim_{h \rightarrow 0} \frac{F(a+h) - F(a)}{h}$$

$$= f'(a) \quad \boxed{\text{E}}$$

$$\textcircled{2} F(x) = \int_1^{1+h} \sqrt{x^5 + 9} dx$$

$$\lim_{h \rightarrow 0} \frac{F(1+h) - F(1)}{h} = f'(1)$$

$$f'(x) = \sqrt{x^5 + 9}$$

$$f'(1) = 3 \quad \boxed{\text{C}}$$

$$\textcircled{3} \text{ I } \checkmark \quad \text{II } \checkmark$$

$$\text{III } \checkmark \quad \frac{1}{b-a} \int_a^b f(x) dx = f(c)$$

$$\boxed{\text{E}}$$

$$\textcircled{4} f(x) = e^{x+e^x} = e^x \cdot e^{e^x}$$

$$\boxed{\text{E}}$$

$$\textcircled{5} \frac{d}{dx} \int_1^{x^2} \sqrt{1+t^3} dt$$

$$= 2x \cdot \sqrt{1+x^6}$$

$$\boxed{\text{A}}$$

$$\textcircled{6} \frac{1}{2} \int_1^3 \frac{1}{x} dx = \frac{1}{2} [\ln(x)]_1^3$$

$$= \frac{1}{2} [\ln 3 - \ln 1] = \frac{1}{2} \ln 3 \quad \boxed{\text{D}}$$

$$\textcircled{7} y = (x^2 + 1)^x$$

$$\ln y = \ln(x^2 + 1)^x$$

$$\ln y = x \cdot \ln(x^2 + 1)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln(x^2 + 1) + x \cdot \frac{1}{x^2 + 1} \cdot 2x$$

$$\frac{dy}{dx} = (x^2 + 1)^x \left[\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right] \quad \boxed{\text{E}}$$

$$\textcircled{8} \boxed{\text{E}}$$

$$\textcircled{9} \boxed{\text{B}}$$

$$\textcircled{10} \int_0^\pi \sin x dx = 2$$

$$\boxed{\text{A}}$$

$$\textcircled{11} \frac{dy}{dx} = \frac{1}{x}, \quad y = \ln x$$

$$\frac{1}{3} \int_1^4 \frac{1}{x} \cdot dx = \frac{1}{3} [\ln(x) - \ln(1)]$$

$$= \frac{1}{3} \ln(2^2) - 0$$

$$= \frac{2}{3} \ln 2 \quad \boxed{\text{C}}$$

12) $f'(x) = (2x-3) \cdot e^{(x^2-3x)^2}$

$f'(x) = 0, x = \frac{3}{2}$ **C**

13) **E** Think MVT for integrals

14) **D** FTC-I

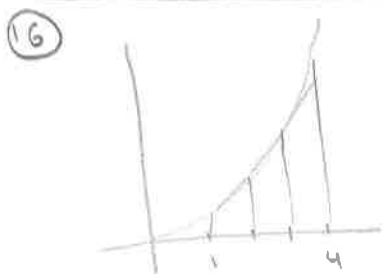
15) $\int_2^3 (e^x - 40x) dx$

$= [2x^4 - 20x^2]_2^3$

$= [2(e^3) - 20(9)] - [2(16) - 20(4)]$

$= 162 - 180 - 32 + 80$

$= 30$ **B**



$T = \frac{1}{2}(1)[1 + 2 \cdot 2^2 + 2 \cdot 3^2 + 4^2]$

$= \frac{1}{2}[1 + 8 + 18 + 16]$

$= \frac{43}{2}$ **C**

17) II only **B**

II - w/ calc

18) $\frac{1}{2} \int_{-1}^1 e^{-x^2} dx = 0.747$ **B**

19) **E**

20) $g'(x) = 100(x^2 - 3x + 2) \cdot e^{-x^2}$
 $= 100(x-1)(x-2)e^{-x^2}$

$100e^{-x^2}$	+	+	+	+
$x-1$	-	+	-	+
$x-2$	-	-	+	+
	(+)		(-)	(+)
	↗		↘	↗

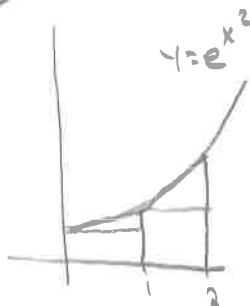
II - only **B**

21) $\int_1^{500} (13^x - 11^x) dx - \int_2^{500} (13^x - 11^x) dx$

$= \int_1^2 (13^x - 11^x) dx = 14.946$

B

22)



Inscribed \Rightarrow LRAM

$A_{LRAM} = 1 + e$

$A_T = \frac{1}{2}(1 + 2e + e^4)$

$A_T - A_{LRAM} = 26.799$

D

24

$$AUC(f) = \frac{1}{4} \int_0^4 x^3 - 4x \, dx$$

$$= 8 \quad \boxed{A}$$

25

\boxed{D}

26 a) $\int_1^2 + \int_2^4 = -4 + 8 = 4$

b) $\int_0^2 - \int_1^2 = 7 - (-4) = 11$

c) $f(x+1)$ is translated left 1 unit

$$\int_1^2 f(x) \, dx = \int_0^1 f(x+1) \, dx$$

$$-4 = \int_0^1 f(x+1) \, dx$$

27

a) $\int_0^2 \sqrt{4-x^2} \, dx$

$$= \frac{1}{4} \pi r^2 = \pi$$

b) $\int_{-2}^2 \sqrt{4-x^2} \, dx$

$$= \frac{1}{2} \pi r^2 = 2\pi$$

c) $\int_{-5}^5 \sqrt{25-x^2} \, dx$

$$= \frac{1}{2} \pi r^2 = \frac{25}{2} \pi$$

27

a) $\int_0^4 \sqrt{16-x^2} \, dx$

$$= \frac{1}{4} \pi r^2 = 4\pi$$

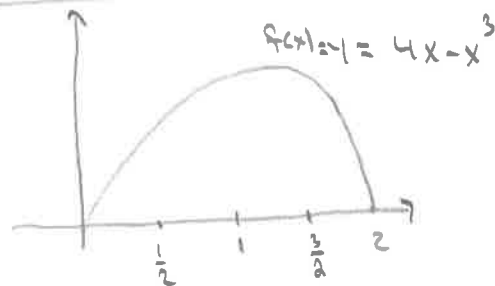
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a) $\int_0^4 x^2 \, dx$

b) $\int_{-1}^3 (3x^2 + 5x + 2) \, dx$

29

a)



b) LRAM = $\frac{1}{2} (f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{2}))$
 $= 3.75$

c) mRAM = $\frac{1}{2} (f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{4}) + f(\frac{7}{4}))$
 $= 4.125$

d) RRAM = $\frac{1}{2} (f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2))$
 $= 3.75$

e) T = $\frac{1}{2} (f(0) + 2 \cdot f(\frac{1}{2}) + 2 \cdot f(1) + 2 \cdot f(\frac{3}{2}) + f(2))$
 $= 3.75$

f) $\int_0^2 (4x - x^3) \, dx = 2x^2 - \frac{1}{4}x^4 \Big|_0^2$
 $= (8 - 4) - (0) = 4$

$$\textcircled{30} A = 2 \cdot \int_0^2 (4x - x^3) dx$$

$$= 8$$

($y = 4x - x^3$ is odd)

$\textcircled{31}$ where is the pond?
(omit)

$$\textcircled{32} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2 x dx$$

$$= -\cot x \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= -\cot \frac{5\pi}{6} + \cot \frac{\pi}{6}$$

$$= \sqrt{3} + \sqrt{3}$$

$$= 2\sqrt{3}$$

$$\textcircled{33} s(t) = \int_0^t f(x) dx$$

$$c) v(5) = s'(5) = f(5) = -2 \text{ cm/sec}$$

$$a) s(0) = \int_0^0 f(x) dx = 0 \text{ cm}$$

$$d) s''(t) = f'(t) = 0$$

$$b) s(3) = \int_0^3 f(x) dx$$

$$t \approx 5 \text{ sec}$$

$$= \frac{1}{2} (2.5 + 1) 3$$

$$e) t \approx 3.5 \text{ sec}$$

$$= 5.25 \text{ cm}$$